Essentials of teacher training sessions with GeoGebra

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Formal requests were recently introduced for integration of ICT in secondary school mathematics. As the main issue, students must develop competence to decide when and how it is appropriate to use available ICT tools and to use them. These new requests put demands on those teachers who have not developed corresponding competencies themselves.

Danish GeoGebra Institute (DGI) designs training sessions for teachers, whith the aim to initiate or support professional development to meet such needs. The design of these sessions does not see mastering the software GeoGebra as an end but as a means for the teacher to reach his teaching goals. Although we see good reasons to choose GeoGebra, other software might serve the same purposes.

The paper presents our model for the design of teacher training sessions aiming at ICT – integration. The model's design is centred on four "essentials", mapping out the influence of ICT for mathematics education. First try out of our model, in a recent project in a small school with ten teachers, is reported in the paper.

1 INTRODUCTION

Formal requests were introduced recently for integrating ICT in lower and upper secondary school mathematics. There is no clear statement, though, which defines the terms ICT and integration. In upper secondary school it is decided at school level, whether the students are provided with/requested to bring with them shared computers, personal laptops or handheld CAS calculators. In some upper secondary schools, tools even in the same class can be mixed between laptops and handheld calculators. Following the Danish tradition for ICT use in secondary school, there are no formal regulations, stating a syllabus or pointing out guidelines for neither the design nor the content of teaching and learning mathematics with ICT; only the goals are stated in broad terms. In upper secondary, the students are not supposed merely to become familiar with the use of one or two tools for standardised problem solving, whereas, ideally, they must end up being able to decide when and how it is appropriate to use ICT tools, to choose amongst available tools and then, subsequently, to use them for solving the problem in question, and to document their reasoning and their tool use. Thus, in our interpretation the students are supposed to reach a meta-perspective on the use of ICT tools. In lower secondary, students' meta-perspective on tool use (at an introductory level) is also prescribed, in accordance with the constructivist's view on learning which is pervading the regulations for both levels. Amongst the mathematics teachers, there is a huge variety in background, interests and formal qualifications for use of ICT – let alone that an unknown part of the mathematics lessons, at least in lower secondary, are taught by non-mathematics teachers. Some teachers have become experts on their own by working with ICT, trying out different settings, preparing teaching materials etc. for maybe ten years or more, some have partly or fully completed formal education in computer science or engineering, whereas others have missed or purposefully neglected even the very few, compulsory or optional courses offered during pre- or inservice teacher training.

2 DANISH GEOGEBRA INSTITUTE'S INITIATIVE

It is no easy task for the teacher to facilitate the students' development of competencies to meet these requests. Rather than providing a series of courses on Derive, Capri, Mathematica, GeoGebra etc., the Danish Geogebra Institute (DGI) attempts to address the issue of competence development in a way that supports the teacher collaboration and their individual use of ICT in teaching. Consequently, we are now developing a design for training sessions, to be offered to groups of mathematics teachers, for example working at the same school. As background for this initiative, we lean upon the reflections about conducting design experiments to support teachers' learning, reported in (Cobb et al. 2009). Cobb et al. address three conceptual challenges: a) to locate the teachers' learning in the institutional setting of the school and district, b) to account for the collective learning of the teacher group and c) to relate teachers' activity in professional development sessions and in the classroom. For the development issues in this article, rather than research issues, bullet c) represents the most relevant challenge. In Cobb's article, the relations between teachers' activity in classroom settings and professional development settings are conceptualized bidirectional, meaning a bidirectional interplay between the teacher's use of artefacts in the two settings rather than a mere two-way movement of for example GeoGebra training sequences and students' tasks. Consequently, the training session must offer more than just an opportunity for the teachers to prepare and discuss teaching materials to be used in their classrooms. Our approach lies somewhere between teaching concrete ICT skills to teachers, and facilitating discussions on important themes regarding ICT in mathematics education. The role of DGI then becomes to initiate collaborative professional development focused on integration of ICT in mathematics teaching in groups of teachers, with one or two experts from DGI as the group's partner.

2.1 Design of training sessions

Our approach was to design an open and flexible scheme for training sessions, meant to be tailored by the DGI-expert to meet the actual needs of the group of teachers in question, without restricting us to a certain level of teaching. Although the actual software used during the sessions is GeoGebra, we do not see the mastering of GeoGebra as an end, but as a means for the teacher to reach his or her teaching goals. The sessions aim to initiate and/or support the teachers' professional development to meet the new demands, following from the introduction of formal requests of ICT integration. Taking a step further from seeing new demands this introduction, though, has huge potentials: We see the ICT integration having a profound impact on school mathematics. The teachers' professional development related to the ICT integration must aim to realise at least some of its potentials for curriculum development, enhanced learning, inclusive teaching, investigative and inquiry learning etc.

2.2 Why structure the field of ICT in mathematics education into "essentials"

As outset for our course development we have structured the field of ICT and mathematics education into a small set of essential issues. The goal was to create a conceptual model that makes the field easier to handle, in that sense we can consider the essential issus as the result of a didactical transposition (Brousseau 1997). More precisely the aim was:

(1) to combine theory and practice

(2) to develop a language to facilitate teachers' and researchers' discussion of the use of ICT in mathematics education, from a concrete as well as more theoretical point of view.

Such ambition can also be found in project as Kompetencies and mathematics learning (Niss et al. 2002) and principles and standards (NCTM 2000). On this background we have identified four *essentials*, characteristic of and necessary for teachers' professional development related to ICT integration in mathematics from our point of view. On the one hand, pointing out distinctiveness of these four essentials results from our attempt to structure the field. On the other hand, the essentials serve as a basis for the DGI experts' collaboration with each group of teachers, on which the sessions can be tailored to meet the individual group's actual needs and interests. Hence, we see the set of essentials as a means to ensure quality in the sessions without loosing the freedom to tailor the individual case of collaboration, taking the variety of school social contexts into account. In the following paragraphs, we present the background for the four and discuss the content of them. The model's designs are centred on the following four essentials:

1) Tool
2) Medium
3) Vehicle for learning
4) Change agent:
a) Rethinking of teaching mathematics
b) New perspectives
c) New content

The essentials are explained and their relevance motivated below, just after we have specified the research question.

2.3 Research question

The research ambition of our project is to test the *concrete essentials* that we have destilated from theory and to test the *overall approach* of structurering a complex research field into more manageable concepts.

This broad ambition is obviously not completely fulfilled in the study described here. The present report describes our first try out of the essentials as a means to structure teaching.

The question for this present paper is; is it possible to identify situations where the essentials:

(1) mediate between theory and practice

(2) support the communication about the use of ICT in mathematics education, in relation to in service education of teachers.

3 POINTING OUT THE ESSENTIALS

What is demanded of the teacher, according to the description above, is to establish a learning environment where students can acquire competence, on the one hand, to use appropriate, available ICT tools for mathematical learning purposes including task- and problem solving and modelling. To fulfil this, obviously, the teacher must be able to use the tool for problem solving and modelling him/her self. Work and study within this area is, in this notion, included in the term *Tool essential*. On the other hand, the student is also supposed to develop competence to judge and make decisions about use of ICT tools, and to document his or her use. To be in front of this, the teacher necessarily has to work and study ICT tool from the perspective of a medium; what happens with mathematics during the transformation into and out from for example the laptop, how could the mathematical meaning be expressed and interpreted in different media, what could a semiotic view add to understanding etc. These issues are included in the term *Medium essential.* To initiate, guide and support the students' tool use for mathematical content learning purposes, the teacher has to work and study ICT tools in mathematics at a meta-level with regard to mathematical learning, which is termed *Vehicle for learning essential.* The fourth, termed *Change Agent essential*, is structured into subsections which may partly overlap the first three essential. In the following we argue, leaning upon the notion of *teachers' forced autonomy*, for seeing Change Agent essential as a characteristic, important for the teachers' professional development. The teacher has to find a way to ensure that the students acquire these new competencies, and to evaluate the process[1]. Like in Skott's discussion of the forced autonomy of mathematics teachers (Skott 2004 p 239), this is a case of *'expected classroom practices and learning outcomes formulated outside the classroom, but there is no set of well-defined methods for the teacher to carry out and only vague hints as to what kind of practice a certain situation may require. '*. Skott argues, based on his study, that the notion of forced autonomy, based on the conceptions of mathematics and mathematical learning, should be extended to encompass not only the roles of the teacher when supporting students' learning in classrooms, but also the multitude of other obligations that emerge in the course of the classroom interactions, which further complicate teacher decision-making. The extended notion of forced autonomy should, according to Skott, serve as a better means for researchers to understand the teachers' role for the enacted curriculum. But also, referring to one of the cases in Skott's study, provide the teacher and his colleagues with an understanding that prevents them from oscillating between facilitating mathematical learning and pursuing broader educational aims. Recent Danish studies of laptop-, computer- and handheld CAS calculator- classes (Andresen 2006) point to a similar complexity of forced, extra obligations, related to different aspects of the introduction of ICT tools into the classroom. Some additional, extra obligations were found which do not relate directly to the introduction of ICT tools. Thus, teamwork and interdisciplinary projects were introduced by formal request simultaneously with the ICT in upper secondary and thereby constitute part of the context. In line with Skott, these factors of influence must be taken into account, additional to conceptions of mathematics and mathematical learning when coping with the teachers' forced autonomy. In our model, the Change Agent essential is meant to capture changes of teaching and learning practice. Consequently, we included into the Change Agent essential a number of relevant changes, experienced by Danish teachers in practice. The changes have induced extra obligations for the teacher, some of them forced and some in the context of teaching experiments or developmental projects. The extra obligations have to be addressed in the training sessions, because they complicate the teacher's decision-making and constitute potential obstacles to the integration of ICT in mathematics. In our design training sessions, which concentrate on the Change Agent essentials take experienced practice as the starting point and connect the experiences with relevant issues and elements from the three other essentials, during the workshops and discussions in the group of teachers. In the following, the content areas of each of the four essentials are briefly outlined, to give an impression of the character of our prospective work.

[1] in Denmark teachers are obliged, yearly, to grade the students' daily work. In upper secondary mathematics, the students daily written and oral s are graded separately

3.1 Tool essential: Generating tools from GeoGebra and other ICT.

In our settings, work and study within the area of tool use takes as its starting point that in itself, GeoGebra does nothing. Though being a necessary prerequisite, teachers' knowledge about GeoGebra's features and facilities as well as the potentials of these in specific mathematical contexts, moreover, will still not be sufficient for the teacher to guide the students' development of the competence required. The teachers need didactical insight into and experience with the generation of a tool from ICT like in this case GeoGebra.

According to the theory of *Instrumental genesis and instrumented techniques,* an artefact, for example GeoGebra, does not in itself serve as a tool for anybody. It becomes useful, and then denoted an "instrument", only after the user's formation of (one or more) mental utilisation scheme(s). Such utilisation schemes connect the artefact with conceptual knowledge and understanding of the way it may be used to solve a given task. Thereby, the utilisation schemes contribute to the formation of instrumented action schemes. So, an instrument consists of the tool, for example a laptop with GeoGebra, the user's mental utilisation schemes and the task or problem to be solved. (Drijvers 2003 p 96-97). The term "instrumental genesis" denotes the process in which the artefact becomes an instrument. (Drijvers & Gravemeijer 2005 pp 165-169). The formation of utilisation schemes and building up instrumented action schemes proceeds through activities in "*The two-sided relationship between tool and learner as a process in which the tool in a manner of speaking shapes the thinking of the learner, but also is shaped by his thinking.*"(Drijvers & Gravemeijer 2005 p 190). In the notion of instrumental genesis, the process directed toward the student is called "instrumentation" and the process directed toward the artefact is called "instrumentalisation", meaning "shaping" and "being shaped". The theory of instrumental genesis builds upon the *scheme concept*, encompassing utilisation schemes and instrumented action schemes introduced by G. Vergnaud as *"..an invariant organisation of activity for a given class of situations. It has an intention and a goal and constitutes a functional dynamic entity. In order to understand its function and dynamic, one has to take into account its components as a whole: goal and subgoals, anticipations, rules of action, of gathering information and exercising control, operational invariants and possibilities of inference within the situation"* (Trouche 2005 p 149) The utilisation schemes are mental and connected with *instrumented techniques* as the external, visible and manifest parts of the instrumented action scheme. The notion of instrumented techniques origins from France: the second part of this term, the concept of "technique", is included in Chevallard's notion of "praxeologies" composed by the four components: i) Type of task, ii) Technique to solve it, iii) Technology to explain and justify the technique and iv)Theory to form the basis for the technological discourse. An "instrumented technique", then, is a *"..set of rules and methods in a technological environment that is used for solving a specific type of problem.*" (Drijvers & Gravemeijer 2005 p 169). An instrumented technique involves conceptual elements as far as the technique reflects the schemes. This leads us to a crucial conclusion: The student's development and use of instrumented techniques cannot be separated from the student's development of those instrumented action schemes, to which the techniques relate, meaning that development of mathematical conceptions cannot be supported by teaching that considers use of technology separate from the student's other activities. In line with this, Jean-Baptiste Lagrange stresses, that *"..the traditional opposition of concepts and skills should be tempered by recognising a technical dimension in mathematical activity, which is not reducible to skills. A cause of misunderstanding is that, at certain moments, a technique can take the form of a skill"* (Lagrange 2005 pp 131-132). The challenge for the teacher is to create a learning environment where GeoGebra can form such technical dimension in the students' mathematical activity. Our rationale for introducing the tool essential, including an introduction to the theory of instrumental genesis besides a hands-on course on GeoGebra in a specific mathematical context is to enable him/her to do it.

3.2 Medium essential: ICT changes the mediation of mathematics

A mathematical computer program such as GeoGebra, is of course a tool that assist individuals in their mathematical activity, but such a program can also be used to store and convey mathematical meaning and to construct mathematical artifacts, we describe this as using GeoGebra as a mathematical medium. One definition of media is as the material substance used for signification (Misfeldt 2008). Following this definition pen and paper is a medium (for instance used for signifying written signs and diagrams), sound is a medium (for instance used for signifying speech and music). We choose to consider a computer with a configuration of programs as one medium. Hence one can discuss the way that this medium supports mathematical meaning expression. How the medium supports expression and conveying meaning is not isolated from the more conceptual aspects of leaning and working with mathematics. The influence of media on the conceptual content of a message has been described in media studies (McLuhan 1964) with a mantra of "the media is the message". The idea is that different media constitutes different ways of expressing oneself – different ways of augmenting human potential. Furthermore the intersection of signification and conception is a persistent theme in mathematics education (Sfard 91, Radford 2002, Duval 2006).This means that choice of medium, with its affordances towards specific modalities, can significantly influence learning of mathematics. Modalities are culturally developed systems of signification, such as speech, writing, painting and cartesian graphs (Kress and Leeuwen 2001). Looking at GeoGebra as a mathematical medium means that we should focus on at least: What kind of "messages" or mathematical artifacts, can you construct? Are GeoGebra especially useable for some types of mathematical meaning expressions or modalities, does the program afford new ways of expression mathematical meaning, and how does Geogebra relate to the existing ways of expressing mathematics? These questions will be explored below.

<u>A medium with mathematical capabilities:</u> Dynamic geometry Systems (including GeoGebra) has the feature that every diagram you create with a DGS respects, if constructed correctly, the mathematical theory. This means that a diagram created with a DGS connects the look and feel of the diagram with the theoretical interpretation of the diagram more easy than a paper based diagram (Laborde 2005).

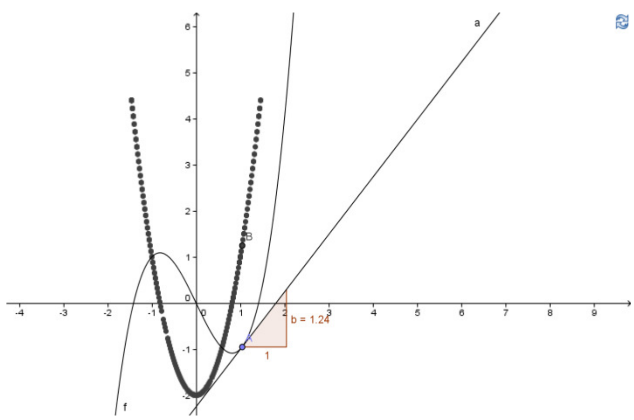
<u>Writing as a challenge:</u> Linear writing is not supported by GeoGebra, but it is easy to export figures to a text processer. Geogebra is not a text processor, and does not serve the purpose of writing mathematics in a linear form. Using a computer based medium (such as a CAS, LaTeX or a formula editor) for writing mathematics persists to be a challenge (Misfeldt 2008).

<u>The applet as a different medium:</u> The possibility of creating small applets provides a different way of producing, retrieving and conveying mathematical knowledge. Java-based applets with mathematical content has existed for more than 10 years as a supplement and alternative to textbook illustrations. GeoGebra provides an easy way to produce such applets. The fact that you can easily, with minimal technical knowledge, produce mathematical applets have at least two consequences for mathematical mediation. One consequence is that teachers have easy access to create dynamic illustrations of the mathematical phenomenon's and problem situations that her class works with. Furthermore these applets can be shared with colleagues, and indeed the GeoGebra Wiki is full of applets made by teachers to support the teaching of a number of topics. Another possible use of GeoGebra could be to have students or pupils create their own applets about topics that they work with. From a student perspective this would mean that the web applet, can be considered as a medium for expressing, not only retrieving, knowledge.

<u>The construction protocol:</u> The construction protocol is a tool that allows us to communicate how we constructed a specific diagram. This can be useful in relation to homework and in other areas where the process is important.

3.3 Vehicle for learning essential: Impact on concepts, reflections, focus…

That ICT changes the way that people can access knowledge and learn is in many ways well established. The possibility of creating ICT based mathematical microworlds is one of the first potentials that have been explored (Papert 1980, Noss and Hoyles 1996). This approach promises a radically new learning process building on the interactive and expressive powers of ICT. Another example is the change of focus from technical details to concepts, processes and relations (Andresen et al. 2004), also described as the CAS potential (Winsløw 2003). Furthermore ICT can increase flexibility of mathematical concepts, gained by repeatedly changing between different representations and perspectives (Andresen 2006). One example of GeoGebra as a "vehicle for learning" can be given looking at the concept of *derivative*. Derivatives is usually considered a hart topic and the introduction of the topic is complex because it involves several new ideas to the students. The idea of a fuction (the derivative) not, apriori, given by an algebraic expression, the idea of tangent, and the idea of limit. Using GeoGebra you are able to introduce the idea of derivative without using the concept of limit:



Figur 1

Looking at the above example, we see one way that ICT can allow a specific focus when introducing the notion of derivative. The above drawing is constructed by placing a point A on the graph of the function *f(x)*=x^3*-2x* and then take the tangent to the graph of *f* GeoGebra allows you to move the point around on the graph of *f*. You can then place a geometrical tangent to the graph through the point A. by defining a point B with the x-coordinate from point A and y-coordinate that is the slope of the tangent through A you can get the shape of the graph of the derivative function, without considering limits at all.

By learning about derivatives without having to care about the notion of limits one can focus on the geometrical construction, and how the derivative acts in a number of examples without facing the hard problem of teaching limits, involving infinity, and in general being considered a very hard topic. Of course this possibility also shows a potential pitfall because the possibility to illustrate the principles in derivative without considering limits, does not mean that the concept of limits is unimportant or should be neglected.

This example illustrates what the notion of Vehicle for learning essentials is meant to cover; the teachers' conscious use of GeoGebra to realise certain potentials for special learning gains linked with specific mathematical topics.

3.4 Change agent essential: The role of ICT for changing the teaching agenda.

The introduction of ICT is not the only change in teaching and learning mathematics nowadays. But rather than seeing the introduction of ICT as a unique happening amongst others, the change agent essential serve to give opportunity for the participating teachers to let new teaching challenges be the starting point for the introduction of GeoGebra into mathematics. For example, the ask for 'inquiry teaching and learning in a constructivist approach' may lead to a GeoGebra training session that concentrate on a workshop working style in primary school, or the request of multi-disciplinary teaching projects in upper secondary may lead to a GeoGebra training session on a suitable subject, maybe even involving teachers from other subjects than mathematics. Danish teachers work with a high degree of professional autonomy and may, most likely, be critical to any seemingly unfounded top-down claim for change of their teaching. So, the formal requests may not be sufficient for some of the teachers to motivate deep changes of practice. On the other hand, even if much research has been done on the use of ICT in mathematics teaching and learning, the outcomes do not give a holistic picture and many questions are not resolved. (Grevholm 2009 p 284). Our rationale for introducing the change agent essential is to support a bottom-up development of teaching practice with ICT. Thereby motivated, we intend to establish a collection of relevant examples of altered teaching practices involving ICT, experienced by teachers in, or outside, recent research-and/or development projects. The collection, then, may serve as a basis for our tailoring of actual training sessions with GeoGebra. These experienced changes may be grouped into three:

a) Rethinking of teaching mathematics: new roles for the teacher in teamwork and in the classroom: Teamwork and New roles for the teacher, directly related to ICT, include change of the teacher's role as an expert but also the obligations to handle students downloading music, technical problems with power supply, sharing of homework tasks, disfunctional groupwork around the computer, focused vs. unfocused student activity, trial and error etc. (Andresen et al. 2004 p 12)

b) New perspectives: modelling aspects, applications, authentic models: These changes may for example appear in connection with the teaching of interdisciplinary projects (which do not necessarily relate to the use of ICT. For examples, see (Andresen and Lindenskov 2008)). According to (Niss, Blum and Galbraith 2007), ICT provides not only increased computational power, but broaden the range of possibilities for approaches to teaching, learning and assessment. But on the other hand, the use of ICT may also bring associated problems and risks, raising questions like for example: "*Will button pressing compromise the thinking and reflection necessary within modelling problems, or can these be enhanced by technology?*" Or "*When does technology potentially kidnap learning possibilities, e.g. by rendering a task trivial, and when can it enrich them?*" (Niss, Blum and Galbraith 2007 p 24). Such questions reflect ongoing discussions to be dealt with as one part of the change agent essential.

c) New content: A number of new topics were introduced in upper secondary during the 'World Class Math and Science' Project 2000 – 2006, which was reported in (Andresen et al. 2004, and on www.matnatverdensklasse.dk (in Danish only)). For example, Numerical solution of differential equations, tests like 'scrambling' and 'bootstrapping', curves like Archimedes' spiral, Cykloide, Astroide, Deltroide etc., regression and others.

4 FIRST TRY OUT

In spring 2009 we got the opportunity to try out the model in the context of a small school-development project in a private school (K-5) with 12 teachers and around 100 pupils. The project, being financed by the school, was designed as a 30 hours course for all the teachers, linked with several group activities embedded in the school's normal life. The project aimed to establish a common platform of mathematics and a shared view on the school's mathematics teaching between the teachers. The aim of the course was the following (our translation) *'..to supply the teachers with basis for, on their own, to develop and design mathematical activities in the classroom, based on insight into theory of mathematics education, on knowledge about concrete examples of activities and teaching materials and based on discussions of didactics and pedagogy in the group.'*

We decided at the meeting to include one session (150 min.) concerning the use of computers/GeoGebra in mathematics, focusing on computer aided or based activities in line with the rest of the project's pedagogic. Our planning of the computer session followed our ideas about tailoring a DGI training session. The basic idea of using the four essential to structure a DGI training sessions is revealed in the following description of our planning of the computer session: We choose to put weight on the V*ehicle for learning essential* and on the *Medium essential* with geometry in focus as the mathematical topic. We chose geometry because it was well suited to GeoGebra and because the teachers had already had a geometry workshop with concrete materials. The content of the computer session was a short power-point introduction to the main geometrical conceptions in primary school (such as to situate, to measure and to construct, distance and shape) with examples of activities with or without the use of computer. Van Hiele's model of levels in geometry (Clements 2003 p152 ff) was presented with examples to give background for the following workshop, and so was each of the conceptions of translations, reflections, rotation, patterns and symmetry. This part prepared for the workshop part of the session, where the teachers went to the computers and were introduced to GeoGebra by a tutorial. The workshop part of the session had tessellations as its subject. We choose the subject tessellations for several reasons: 1) some of the pupils had already experiences with tessellation patterns from the painting of printed sheets, 2) it offers good opportunities for creative work and play with the mathematical conceptions reflection, translation, symmetry etc. 3) it offers good opportunities for interaction with the computer, especially with GeoGebra 4) it is useful to create artefacts in the form of printed sheets. Bullet 2) and 3) relates directly to the *Vehicle for learning essential*, whereas bullet 4) relates to the *medium essential.*

During the workshop part of the session, we had informal talk and discussions with the teachers about issues, captured by these two essentials. For example, the teachers were provided with a schematic overview of tessellation with polygons. These materials served as a basis for them to consider what kind of questions and open ended tasks they could ask the pupils, and for which learning purposes. In parallel, they got familiar themselves, not only with GeoGebra, but also with the subject. One example of a *medium essential* issue was the importance of keeping hard copies, in the form of paper sheets, of the pupils' own patterns and tessellations which was stressed in our informal talk with the teachers about the actual planning of lessons. This is an issue of transforming the artefact (the pattern or tessellating) from being in the computer into a more tangible, permanent and public, visible form. Regarding the vehicle for learning essential, the teachers needed strong guidance to experience and reflect on the potential of learning about tessellations with GeoGebra. There can be various reasons for this, some having to do with our instruction and the teachers' knowledge about the underlying concepts of symmetry. In contrast, the medium essential was more directly appealing to the teachers; they could see how symmetries and tessellations could be inspiring topics to explore from an aesthetic point of view. This could be due to a combination of the teachers' main topics not being mathematics, and the aesthetic nature of tessellations and symmetry.

5 RESULTS

We have seen that discerning between the medium essential and the vehicle for learning essential was productive for mediating between theory and practice and for support of the communication. These two concepts allowed us to address more precisely an element of ICT use for mathematics education that resonated with the teachers interest and needs.

In the case repported here we saw that the medium essential resonated with the target groups interests and ambition for their teaching. The articulation of the medium essential allowed the teachers to discuss tesselations and other mathematical products in detail, and from a theoretical - didactical point of view. It became clear that the teachers where interested in discussing how to support that the students used ICT to create mathematical products and share them with other students.

Talking about the "medium essential" supported the development of that discussion and made it legitimate.

6 CONCLUSION

Taking the new formal requests of introduction of ICT into mathematics teaching as our starting point, this project aims to empower the interplay between theory and practice and to develop a language for professional discussions of mathematics teaching. Our structuring the area into four main essentials: tool, medium, vehicle for learning and change agent, leans upon an extended conception of teachers' forced autonomy, taking not only mathematical conceptions and meta mathematical conceptions into account but also changes in teaching practice. The essentials serve as a means to design training sessions. One try out of a design based on the essentials was reported in the paper, showing that it was productive for mediating between theory and practice and for support of the communication to discern between at least two of the essentials.

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